

Specificity of Phase Transition of Quasi-Spin System in Two-Quantum Exchange with Thermostat

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July 7, 2008

Abstract

The quantum phase transition of the system of N radiators in case of two-photon exchange interaction with cavity electromagnetic field is considered. It is shown that for this system the atom-field exchange integral increases with the increase of temperature. This effect generates peculiarities in phase transition picture. The process of nonlinear exchange between the quasi-spins leads to the increase of critical temperature as compared to traditional second order phase transition of quasi-spin system.

Keywords: Phase transition; Two-quantum interaction; Critical temperature; Quasi-spin system

PACS: 05.70.Fh; 05.30.-d

The cooperative emission phenomenon for dipole-forbidden transitions of inverted system of radiators can be observed in the processes of two-photon spontaneous emission in free space [1] or cavity [2]. It is important from the physical point of view to study the cooperative phenomena in a larger aspect of statistical physics. A great interest in the investigation of statistical and thermodynamical properties of quantum systems represents the existence of phase transition such as the order-disorder transition [3]. For example a radiative system like Dicke model [4] exhibits phase transition from the phase where the radiators

emit non-correlated to the phase where cooperativity between the radiators is established in such a way that the intensity of emission becomes proportional to the square number of radiators, named as super-radiance. In recent years, the study of the phenomena which appear in physical systems when the electronic or atomic subsystems are in non-linear interaction with the thermostat (large subsystem) is of particular interest. For example, the non-linear mechanism of superconductivity [5, 6], bipolaron effects [7], two-photon emission [1,2,8] and other non-linear phenomena. Very recently, authors in [6] obtained some interesting results for the feature of phase transition for superconductivity in the case when a non-linear exchange interaction of the electronic subsystem with the phonon subsystem exists. Therefore this paper is devoted to the non-traditional behavior of order parameter for two-phonon superconductivity thus stating that the enhancing of the critical temperature becomes possible in comparison with usual BCS model. So we concluded that the existence of dependence of two-phonon exchange integral on temperature by means of the average number of phonons changes essentially the feature of phase transition and consequently the value of critical temperature with possibility to enhance this. The peculiarities of two-photon exchanges between two subsystems was discussed in papers [8, 9]. In paper [9] it was shown that in special conditions the two-photon quantum oscillator can turn from incoherent to coherent stimulated emission near the threshold in the same fashion as the one-photon laser. Thus were obtained special ignition and evolution processes (similar to phase transition, see [10]) of two-photon laser from the incoherent to stable coherent emission in the case when the two-photon losses in multi-mode cavity prevail over the traditional one-photon losses in the resonator mirrors.

In this paper we are interested to study the cooperative phase-transition of the system of N radiators that interact with the cavity thermostat via the two-quantum exchange processes. The two-photon process is possible when the one-photon interaction with the thermostat is forbidden. Such a possibility can be realized in microcavities with Rydberg atoms similarly to the experiments with two-photon maser effect described in [11]. For a large number of atoms in the cavity the phase transition through the two-photon interaction with the cavity modes can be possible. In this case the anomalous behavior of temperature dependence of the super-radiant order parameter (e.g. atomic polarization) is investigated. As the two-photon exchange integral between the radiators strongly depends on the temperature via the average number of quanta similar to the two-phonon superconductivity [6], so that the increasing of the order parameter with temperature in quasi-spin systems is observed. This effect leads to the increase of value of the critical temperature as compared to the traditional phase transition of quasi-spin system in which the exchange integral between the spins does not depend on the temperature.

Let us consider an ensemble of radiators in the micro-cavity that enters into two-photon resonance with cavity electromagnetic field. In bad cavity limits when the two-photon spontaneous emission rate is less than the losses of cavity we can adiabatically eliminate the EMF operators and obtain the master-equation for the atomic system discussed in [12].

$$\begin{aligned}
\dot{\rho} = & i(\omega_{21} - 2\bar{n}^2\chi) [\rho, R_z] - i(1 + 2\bar{n})\chi [\rho, R^+R^-] \\
& - (1 + \bar{n})^2 \gamma (\rho R^+R^- - 2R^- \rho R^+ + R^+R^- \rho) \\
& - \bar{n}^2 \gamma (\rho R^-R^+ - 2R^+ \rho R^- + R^-R^+ \rho).
\end{aligned} \tag{1}$$

Here the following parameters are defined: ω_{21} is the atom transition frequency, \bar{n} is the average number of emitted photons in the cavity. The parameter $\chi = \frac{A(k)}{\hbar^4} \frac{2\omega_k - \omega_{21}}{(\omega_{21} - 2\omega_k)^2 + 4\Gamma^2}$ describes the cooperative atom-atom interaction through the cavity EMF vacuum and $\gamma = \frac{A(k)}{\hbar^4} \frac{2\Gamma}{(\omega_{21} - 2\omega_k)^2 + 4\Gamma^2}$ is a spontaneous emission rate of single atom in the cavity, where $A(k)$ is considered for the scheme of transitions taken into account in [12] in the form of

$$A(k) = \left[\sum_{\alpha} \frac{(\mathbf{g}_k \mathbf{d}_{\alpha 1})(\mathbf{g}_k \mathbf{d}_{\alpha 2})(\omega_{\alpha 1} - \omega_{2\alpha})}{(\omega_{2\alpha} - \omega_k)(\omega_{\alpha 1} - \omega_k)} \right]^2.$$

Here $\mathbf{d}_{\nu\beta}$ is the moment of dipolar transition between the virtual states $|\alpha\rangle$ and states $|\beta\rangle$ ($\beta = 1, 2$); $\mathbf{g}_k = \sqrt{2\pi\hbar\omega_k/V}\mathbf{e}_{\lambda}$ is atom-cavity interaction constant, where \mathbf{e}_{λ} is polarization and V is volume of micro-cavity. In Eq.(1) are considered the following cooperative quasi-spin operators R^+ , R^- and R_z are defined like in [13] which are obeying the commutation relations: $[R^+, R^-] = 2R_z$ and $[R_z, R^{\pm}] = \pm R^{\pm}$.

From the above master equation it is observed that for a larger value of the detuning $\delta = 2\omega_k - \omega_{21}$ comparatively to the cavity damping rate Γ , the value of two-photon decay rate, γ , is less than the absolute value of exchange integral, χ . In this case $\chi/\gamma \gg 1$ and we resume that the Hamiltonian part which describes the two-photon exchange interaction between the radiators predominates the losses part in the master equation (1). The effective Hamiltonian that describes this interaction can be expressed as

$$H^{\text{eff}} = \omega R_z - \lambda R^+ R^-, \tag{2}$$

where the following notations are introduced : $\omega = \hbar(\omega_{21} - 2\bar{n}^2\chi)$ and $\lambda = \hbar\chi(1 + 2\bar{n})$. Here λ represents the energetic exchange integral between the radiators and it is observed that this parameter depends on the average number of emitted photons in the cavity. We emphasize that in comparison with phase transition of traditional quasi-spin system like Dicke model in our case the exchange integral depends on temperature through the number of photons, $\bar{n} = (\exp[\hbar\omega_k/\theta] - 1)^{-1}$, $\theta = \kappa T$, where κ is Boltzmann constant. Using the conservation of Bloch-vector at low temperature $R^2 = R_z^2 - R_z + R^+R^-$ (where R is a constant), we can express the Hamiltonian (2) through the operator R_z

$$H^{\text{eff}} = \varpi R_z - \lambda(R + R_z)(R - R_z), \tag{3}$$

where $\varpi = \omega - \lambda = \hbar[\omega_{21} - \chi - 2\chi\bar{n}(1 + \bar{n})]$. Thus from the condition of existence of the minimum value of the Hamiltonian on the operator R_z , the value \tilde{R}_z can be obtained in which the system relaxes

$$\tilde{R}_z = -\frac{\varpi}{2\lambda} \quad (4)$$

and therefore the value of parameter λ must be positive from the condition of existence of absolute minimum value of the Hamiltonian. This is possible when the detuning δ takes positive values. As the ratio ϖ/λ takes arbitrary values the restriction of values for parameter \tilde{R}_z can be obtained taking into account the condition of conservation of Bloch-vector, where $R_{\max}^2 = j(j+1)$. From this expression follows that R_z must take the values between $-j$ and j , i.e. $-j < \langle R_z \rangle < j$. Here $j = N/2$ represents the maximum value of Bloch-vector, N is the number of atoms in the cavity. Thus, for the case of a large number of radiators from the inequality of R_z and Exp. (4) it follows that the two-photon exchange integral must satisfy the inequality $-2j\lambda < \varpi < 2j\lambda$. The fact that $\lambda > 0$ is very important, because in this case the last term in the effective Hamiltonian (2) will imply the tendency to the order phase of the system of radiators. Thus, for the case when $\lambda < 0$ the phase transition is not possible.

Let us consider that $R^+R^- = \sum \sum_{k,l} \sigma_k^+ \sigma_l^- = (R_z + j) + \sum \sum_{k \neq l} \sigma_k^+ \sigma_l^-$, where σ_k^\pm are the Pauli spin operators. Here the first term describes the interaction of quasi-spins with self radiation field and is similar to traditional Lamb shift. The last term describes the interaction of the k quasi-spin with the field of other l quasi-spins, while $k \neq l$. For large value of j this term can be approximated in the thermodynamic limit (when $\lim(N/V) = \text{const}$, while cavity volume is greater than the volume per atom in the cavity) in the form of

$$\sum_{l=1}^N \sum_{k=1, k \neq l}^N \sigma_k^+ \sigma_l^- = \sqrt{N} \sum_{m=1}^N (C\sigma_m^+ + C^*\sigma_m^-) - NCC^*, \quad (5)$$

so that the quadratic Hamiltonian (2) can be approximated in the manner of Bogolyubov method [14] by a linear model Hamiltonian.

In the mean field approximation theory the atoms in the system can be regarded as identical radiators so that the spin operators σ_m^\pm do not depend on the atomic label. Thus the Hamiltonian (2) according to expression (5) takes the following form

$$H_A = N\varpi J_z - N\lambda (CJ^+ + C^*J^-) + N\lambda CC^*, \quad (6)$$

where the number-operator $C \equiv \langle J^- \rangle$ plays the role of order parameter of the system of radiators; here new operators $J^\pm = R^\pm/\sqrt{N}$ and $J_z = R_z/N$ are defined which obey the commutation relations similar to the operators for radiators.

Further we would like to describe the feature of phase transition of quasi-spin system from uncorrelated to correlated state where the cooperativity between the radiators is established as temperature dependence. The expression for the order parameter, C , can be found from the condition of absolute minimum of free energy since we take into account the thermodynamic analysis of the system. As was shown in the past in many papers related to the subject of

phase transition for Dicke model [4] the free energy for the system described by Hamiltonian (2) coincides in the thermodynamic limit with the free energy calculated by model Hamiltonian (6) choosing the values of parameters C and C^* from the condition of absolute minimum of free energy. Denoting the point of minimum by \tilde{C} we have $\lim_{N \rightarrow \infty} F_N[H^{\text{eff}}] = F_\infty[H_A(\tilde{C})]$ or in other words

$$\begin{aligned} F[H_A(C)] &= -\theta \log (\text{Tr} \exp [-H_A/\theta]) \\ &= -N\theta \log \left(2 \cosh \frac{\sqrt{\varpi^2 + 4\lambda^2 |C|^2}}{2\theta} \right) + \lambda N |C|^2. \end{aligned} \quad (7)$$

Thus on condition that $(\partial F/\partial C)_{C=\tilde{C}} = 0$, the value of order parameter, \tilde{C} , can be found by solving the following transcendental equation

$$|\tilde{C}| = \frac{\lambda |\tilde{C}|}{\sqrt{\varpi^2 + 4\lambda^2 |\tilde{C}|^2}} \tanh \frac{\sqrt{\varpi^2 + 4\lambda^2 |\tilde{C}|^2}}{2\theta} \quad (8)$$

The critical temperature is calculated from the consideration that for $\theta = \theta_{\text{cr}}$, the order parameter $|\tilde{C}| = 0$ and the value of this temperature is found by solving the equation

$$\tanh \frac{\varpi_{\text{cr}}}{2\theta_{\text{cr}}} = \frac{\varpi_{\text{cr}}}{\lambda_{\text{cr}}}, \quad (9)$$

where ϖ and λ from the above definitions are functions of temperature and thus will depend on the critical temperature. Hence we see that from Eq. (9) it is not possible to find the explicit expression for critical temperature but the equation can be solved numerically. As the function hyperbolic tangent takes the values between -1 and 1 one can find the relations between χ and ω_{21} which satisfy $2\bar{n}_{\text{cr}}^2 < \omega_{21}/\chi < 2(1 + \bar{n}_{\text{cr}})^2$ and thus the critical temperature is in strong connection with values of parameters χ and ω_{21} .

Exp.(8) clearly defines the important role of the two-photon exchange integral, λ , under the temperature dependence of order parameter, C . The intrinsic temperature dependence of the physical quantities λ and ϖ strongly influences the behavior of phase transition and new features are manifested in comparison with traditional phase transition in quasi-spin systems when exchange integral is constant. This peculiarity is observed from Fig.(1) in which we compare the traditional behavior of second order phase transition with temperature dependence of the order parameter discussed in our model, see Exp.(8). It is an interesting fact that, in the case of our model the temperature evolution of the order parameter in dependence with the value of the parameter χ/ω_{21} differs substantially from the evolution of the order parameter calculated for traditional quasi-spin model which is obtained from Exp.(8) omitting the term which is proportional to the mean number of photons, \bar{n} . Let us consider, for example, the case when

$\chi/\omega_{21} = 0.5$ for which the phase transition in the usual quasi-spin model is absent. As follows from the numerical results of Eq.(8), the phase transition is possible in this case, plotted in Fig. (1a). From this figure we observe that the order parameter appears from vacuum fluctuations of cavity field which increases with the increase of the temperature. Achieving the maximum value the order parameter decreases to zero value in critical point computed from Exp.(9). The small enhancing of value of parameter, $0.5 < \chi/\omega_{21} < 0.6$, leads to the appearance of phase transition in traditional quasi-spin model which exhibits a critical temperature to a lesser value in comparison with phase transition in case of proposed model (see Fig.1b, c). Further enhancing of value of χ/ω_{21} will evidence inessential differences between the compared models and is of no interest for numerical and graphical interpretation.

Another interesting result refers to the mean value of atomic population, $\langle R_z \rangle$, which can be calculated by the standard definition, $\langle R_z \rangle = \text{Tr}\{R_z \rho\}$, or using the expression of free energy the following relation $\partial F[H_A]/\partial \tilde{\omega} = -N \text{Tr}\{\dots J_z\} / \text{Tr}\{\dots\} = N \langle J_z \rangle = \langle R_z \rangle$ is observed, consequently obtaining

$$\langle R_z \rangle = -\frac{1}{2} \frac{\varpi}{\sqrt{\varpi^2 + 4\lambda^2 |\tilde{C}|^2}} \tanh \frac{\sqrt{\varpi^2 + 4\lambda^2 |\tilde{C}|^2}}{2\theta} \quad (10)$$

This value can be compared with the value calculated by Exp.(4) in order to appreciate the differences between the results obtained using the exact effective Hamiltonian (2) and the asymptotic Hamiltonian (6).

From Fig.2 we see that values of atomic population calculated in two different manners coincide in the domain of temperatures close to the critical temperature. This result confirms the accuracy of above calculations where the asymptotic Hamiltonian (6) was considered.

In this paper we discussed the contribution of two-photon exchange mechanism between the quasi-spins to the behavior of phase transition of such system. Therefore obtaining an intrinsic temperature dependence of two-photon exchange integral, the radiator system will exhibit a different fashion of phase transition in comparison with traditional second order phase transition of spin systems. The anomalous temperature dependence of order parameter in the case of proposed model is influenced by the appearance in the expressions of energy, ω , and exchange integral, λ , of additional terms which are proportional to the mean number of photons. From the numerical simulation we obtained that the critical temperature can be enhanced taking into account the non-linear exchange process between the radiators in cavity. This increase of critical temperature depends on the ratio between the parameter of atom-atom interaction, χ , and atom transition frequency, ω_{21} . The calculation of average value of atomic population in two different ways emphasizes the agreement between the initial effective Hamiltonian (2) and approximation Hamiltonian (6) taken in thermodynamical limit. Hence we conclude that the model Hamiltonian (6) describes accurately the behavior of phase transition of quasi-spin system with

two-photon exchange interaction where a non-traditional temperature dependence of the order parameter is manifested.

The discussed problem may be regarded as being of academic interest due to the fact that such temperature dependence can be realized in more complicated non-linear exchanges between the spins through the thermostat. On the basis of two-quantum exchange mechanism between the quasi-spins, in this paper, we pay attention to the possibility of increasing the correlation with temperature in the system.

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Figure Captions

Fig.1 The temperature dependence of order parameter for different values of ratio χ/ω_{21} : a) $\chi/\omega_{21} = 0.5$; b) $\chi/\omega_{21} = 0.51$ and c) $\chi/\omega_{21} = 0.6$. Here the temperature is represented in relative units $T/T_{\text{cr}}^{\text{max}}$, where $T_{\text{cr}}^{\text{max}}$ corresponds to the critical temperature in phase transition of proposed model. The dotted line corresponds to traditional phase transition of a quasi-spin system with constant exchange integral, λ (for example Dicke model) and full line evidences the phase transition in the case of the proposed model.

Fig.2 The temperature dependencies of the average value of atomic population calculated using Exp.(4) - dotted line, and by Exp.(10) - solid line, for case $\chi/\omega_{21} = 0.6$.

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